

New schemes for manipulating quantum states using a Kerr cell

Marco Genovese and C.Novero

Istituto Elettrotecnico Nazionale Galileo Ferraris, Str. delle Cacce 91, I-10135 Torino

Recently, Quantum Non Demolition (QND) detection of a single photon has become possible [1]. This is due to the discovery of new materials with very high Kerr coupling as the Quantum Coherent Atomic Systems (QCAS) [2,1] and the Bose-Einstein condensate [3]. These great technical improvements could allow the realisation of small Kerr cells, capable of large phase shift, even with a low-intensity probe. This recent development has a large relevance for application to quantum information theory and foundations of Quantum Mechanics. For example, it has lead to the proposal of schemes for complete teleportation [4] and for realising quantum gates [4,5]. In this proceeding we describe some other proposals of application of this device, as the fast modulation of quantum interference [6], the generation of optical Schrödinger cats [7] and of GHZ states and the realisation of translucent eavesdropping.

Let us consider first a scheme for realising a fast modulation of quantum interference. A "signal" photon enters a Mach-Zender interferometer through a Beam-Splitter (BS I) from port 1; assuming a 50% BS (the treatment of the non 50% case is a trivial extension) one has after the BS

$$a_2 = \frac{a_1 + ia_0}{\sqrt{2}} \quad a_3 = \frac{a_0 + ia_1}{\sqrt{2}} \quad (1)$$

A probe laser crosses the Kerr cell on the third arm acquiring a phase which in principle will be measurable, providing *welcher Weg* information. The usual treatment of Kerr interaction gives after recombination on Beam Splitter II:

$$\langle \Psi | n_4 | \Psi \rangle = \frac{1}{2} \left[1 - \exp[-2|\nu|^2 \sin^2(\chi T)] \cos[(\omega_s + \chi_s)T + \Theta + |\nu|^2 \sin(2\chi T)] \right] \quad (2)$$

where Θ takes into account different lengths of arms 2 and 3 and could be varied interposing a variable phase shift on one of the interferometer arms. χ_s and χ denote the self- and the

cross- Kerr couplings respectively and n_3 and n_p are the photon number operators on the arm 3 and for the probe, which is assumed to be described by a coherent field $|\nu\rangle$.

To this point, the usual treatment [1] of QND ideal *welcher Weg* experiment has been considered, with the well known result that the probe laser acquires a phase which permits, by a homodyne measurement, the identification of the path followed by the signal photon. The signal-to-noise ratio in the homodyne measurement, given by $R = 4|\nu| \sin(\chi T)$, is directly related to the suppression of interference, whose visibility is $\exp(-R^2/8)$.

Let us now consider the insertion of a second Kerr cell on the 2^{nd} arm and let us assume that the interaction time between the probe and signal fields in this cell is T' . We now have:

$$\langle \Psi | n_4 | \Psi \rangle = \frac{1}{4} \langle \Psi | n_1 [2 - (\exp[-i(\Theta + \beta(T - T'))] + \exp[i(\Theta + \beta(T - T'))])] | \Psi \rangle \quad (3)$$

where $\beta = (\omega_s + \chi_s/2 + \chi_s n_s + 2\chi n_p)$. If we choose the Kerr cells so that $T = T'$, the phase into the probe due to the photon in path 3 or 2 would be the same and the interference pattern $\frac{1 - \cos(\Theta)}{2}$ is recovered for the signal field. On the other hand, if one considers the case where the distance between the two Kerr cells is larger than the coherence length of the probe laser the two paths will still be distinguishable and interference will be lost. Quantum interference can thus be regulated, changing the coherence length of the laser before injecting it into the first Kerr cell. The observation of this effect represents a very good and illustrative example of the effect of disappearance of quantum interference when *welcher Weg* information is obtained and of the effect of erasing this information.

Let us now substitute the first beam splitter with a Polarising-Beam-Splitter I (PBS I) and let us suppose that the entering photon is in a superposition of vertical (V) and horizontal (H) polarisation, which will take different paths, for example the vertical one will follow path 2 and the horizontal path 3. As before, a probe laser crosses the Kerr cell on the arm 3 acquiring a phase or not according if the photon crosses or not the cell. Thus the entangled state:

$$|\Psi\rangle = \frac{|H\rangle|\nu'\rangle + |V\rangle|\nu\rangle}{\sqrt{2}} \quad (4)$$

is generated, where the coherent state $|\nu'\rangle$ differs in phase from $|\nu\rangle$.

The two signal photon paths are then recombined on a second beam splitter (BSII) and a polarisation measurement is performed on this photon on the base at 45° . This is the conditional measurement producing the Schrödinger cat: if the signal photon is found to have a 45° (135°) polarisation, the coherent state is projected into the superposition

$$|\psi_{+(-)}\rangle = \frac{|\nu'\rangle + (-)|\nu\rangle}{\sqrt{2}} \quad (5)$$

A superposition of two "many photons" states is thus obtained.

The one photon signal state can be easily produced, for example, using parametric down conversion (PDC) in a non-linear crystal. In this case the second photon of the down-converted pair can be used as trigger. Furthermore, if the same pulsed laser is used both for pumping the crystal and for the Kerr effect, one can easily obtain a good timing for the crossing of the Kerr cell for the signal photon and the coherent state. A main advantage of the present configuration respect to the Schrödinger cat generation using cavities is that the coherent states superposition travels in air between the Kerr cell and the detection apparatus allowing a much longer time before decoherence takes over.

For identifying the macroscopic superposition one can look for a negative part of the Wigner function, reconstructed by tomographic techniques [10]. We have performed a numerical simulation keeping into account the deterioration due to errors: our results show that even with 25%, or larger, errors on the homodyne measurement the cat can be easily identified.

The use as input of a photon from PDC in a Kerr cell allows also the creation of a three photons GHZ entangled state, which is one of the three elements necessary for realising an optical quantum computer [8], together with single qubit operations, which are easily implemented, and teleportation. For what concerns this last, a description of a scheme performing this operation using a Kerr cell appears in Ref. [4]. Using the same polarisation dependence of the Kerr interaction of Ref. [4], where there is no effect except when both the photons interacting in the Kerr cell have vertical polarization ($|V\rangle|V\rangle \rightarrow |V\rangle|V\rangle e^{i\phi}$), a GHZ state can be generated by the interaction of an entangled pair of photons with a

third one. Let us assume, for example, of having generated the entangled state [9]: $|\Phi^+\rangle = |H\rangle|H\rangle + |V\rangle|V\rangle/\sqrt{2}$. A simple calculation shows that the interaction in the Kerr medium of the second photon of the pair with a third photon polarised at 45° (denoted by $|45\rangle$), whilst its orthogonal state is $|135\rangle$, is, for a phase shift $\phi = \pi/2$, the GHZ state:

$$|\Psi_{GHZ}\rangle = \frac{|H\rangle|H\rangle|45\rangle + |V\rangle|V\rangle|135\rangle}{\sqrt{2}}. \quad (6)$$

Analogous results are easily derived for the other three Bell states.

Finally, let us notice that such an apparatus can also be used for performing translucent eavesdropping on a quantum channel where polarisation is used for distinguishing qubits. In this case the input port of the first beam splitter is fed with a single photon of vertical polarisation, which splits on the two interferometer arms. On arm 1 it interacts with the transmitted qubit inside the Kerr cell (with the same polarisation dependence as before). Let us suppose that the transmitted qubit is in the general form $|u\rangle = \cos(\theta)|H\rangle + \sin(\theta)|V\rangle$, denoting the probe photon with $|p\rangle$, the final state is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left[\cos(\theta)|H\rangle|p\rangle_1 + \sin(\theta)|V\rangle|p\rangle_1 e^{i\phi} + i|u\rangle|p\rangle_2 \right] \quad (7)$$

where the suffixes after the probe $|p\rangle$ denotes the path followed and where we have considered a 50 % BS (which gives Eve the largest information on Alice-Bob transmission). We have thus obtained the desired entanglement between the probe and the transmitted qubit, which can be used for translucent or coherent eavesdropping.

[1] B.C. Sanders and G.J. Milburn, Phys. Rev. A 39 (1989) 694. Q.A. Turchette et al., Phys. Rev. Lett. 75 (1995) 4710; H. Schmidt and A. Imamoglu, Opt. Lett. 21 (1996) 1936; A. Imamoglu et al., Phys. Rev. Lett. 79 (1997) 1467.

[2] U. Rathe et al., Phys. Rev. A 47 (1993) 4994; M.M. Kash et al., Phys. Rev. Lett. 82 (1999) 5229.

- [3] L. Vestergaard Hau et al., Nature 397 (1999) 594. S.E. Harris and L.V. Hau, Phys. Rev. Lett. 82 (1999) 4611.
- [4] D. Vitali et al., Phys. Rev. Lett. 85 (2000) 445.
- [5] G. M. D'Ariano et al., Fort. Phys. **48**, 573 (2000).
- [6] M. Genovese and C. Novero, Phys. Rev. A 61 032102 (2000).
- [7] M. Genovese and C. Novero, Phys. Lett. A 271 (2000) 48.
- [8] D. Gottesman and I.I. Chuang, quant-ph 9908010.
- [9] G. Brida, M. Genovese, C. Novero and E. Predazzi, Phys. Lett. A 268 (2000) 12 and these proceedings and ref.s therein.
- [10] G.M. D'Ariano et al., Phys. Rev. A 50 (1994) 4298, Nuov. Cim. 110B (1995) 237.